Direct Derivative Measurements in the Presence of Sting Plunging

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This paper presents a review and extension of methods for determining the effects of sting oscillation on the measurement of dynamic derivatives due to oscillatory pitching. It is shown that two of the methods based on linear models will reduce to the same result, applicable to low-lift configurations. The location of the effective axis of rotation is determined for a model executing planar oscillations in two degrees of freedom. The equations for the effect of plunging on the derivatives can be simplified by relating the sting deflection parameters to the coordinate of this axis. The implications for analysis of sting plunging oscillation in dynamic tests of aircraft models at high angles of attack are also discussed. The effective rotation center could be determined by a simple method, and the associated effects evaluated, in pitch-oscillation tests of the Standard Dynamics Model at angles up to 45 deg. The axis shift was found to produce the main effect on the measured data.

Nomenclature

$\bar{c}$ = reference length
C_L = lift coefficient
$C_{L\alpha}$ = $\partial C_L/\partial\alpha$
C_{Lq} = $\partial C_L/\partial(q\bar{c}/V_\infty)$
C_m = pitching-moment coefficient
C_{me} = $M_e/(q_\infty S\bar{c})$
C_{mq} = $\partial C_m/\partial(q\bar{c}/V_\infty)$
$C_{m\alpha}$ = $\partial C_m/\partial\alpha$
$C_{m\dot{\alpha}}$ = $\partial C_m/\partial(\dot{\alpha}\bar{c}/V_\infty)$
D, D_z = linear damping constants
I_y = pitch moment of inertia
K, K_z = linear spring constants
m = model mass or effective model and balance mass
M = aerodynamic pitching moment
M_e = harmonic excitation moment
M_p = moment applied to sting at the pivot
M_s = moment measured by sting bridge
q = body axis pitch rate
q_∞ = dynamic pressure
S = reference area
t = time
V_∞ = freestream velocity
x, z = inertial reference system (Fig. 1 or 3)
x' = axial body coordinate (Fig. 1)
z = wind-fixed pivot coordinate (Fig. 1)
Z = aerodynamic normal force
α = angle of attack
$\bar{\alpha}$ = mean angle of attack
δ = increment
δ_{z0} = structural plunge damping, $=\bar{c}D_z/(V_\infty m)$
$\delta_{\theta 0}$ = structural pitch damping, $=\bar{c}D/(V_\infty I_y)$
Δ = amplitude
ζ = $z/\bar{c}$
θ = pitch perturbation angle
κ = $[I_y/(m\bar{c}^2)]^{1/2}$
μ = $m/(\rho_\infty S\bar{c})$
ξ = $x'/\bar{c}$
ρ_∞ = freestream density
τ = $(V_\infty/\bar{c})t$
Φ = phase angle between excitation moment and pitch oscillation
Φ_z = phase angle between translational and rotational motions
ω = angular velocity
$\bar{\omega}$ = reduced frequency, $=\omega\bar{c}/V_\infty$
$\bar{\omega}_{z0}$ = natural frequency in pure translation, $=(\bar{c}/V_\infty)\ (K_z/m)^{1/2}$
$\bar{\omega}_{\theta 0}$ = natural frequency in pure pitching, $=(\bar{c}/V_\infty)\ (K/I_y)^{1/2}$
Ω = frequency dependence of structural damping

Subscripts

a = accelerometer
c = center of rotation
cg = center of mass
f = pivot or flexure
off = wind off
on = wind on
z = translation
0 = single degree of freedom
θ = pitch

Superscripts

$(\dot{\ })$ = partial differentiation with respect to time
$(\bar{\ })$ = fixed-axis derivative, where appropriate
$(\tilde{\ })$ = effective
$(\ ')$ = body axes

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Introduction

THE influence of the motion of a model support system is always present to some extent in dynamic stability tests of captive models in wind tunnels. The effects of sting translational motion on measured stability derivatives are manifested in three different ways:

1) The reactions from which the moment derivatives are obtained contain translational effects.

2) The location of the axis of oscillation is determined by the plunge amplitude.

3) The model angular orientation changes with sting deflection.

Unless these effects are accounted for in the data reduction procedure, the quantities determined cannot be regarded as fixed-axis derivatives. Therefore, the objective is to determine the true single-degree-of-freedom (DOF) oscillatory data in the presence of sting motion.

It has been demonstrated that appreciable errors may be present in pitch damping derivatives determined using sting-supported models if the effects of sting plunging oscillation are not accounted for[1,2]. This is especially true in tests involving bulbous-based,[3] or short, blunt bodies, where very thin stings have to be used to minimize the aerodynamic sting interference, and in high-α tests of aircraft configurations, where significant sting deflections are unavoidable. In many cases, it may, in fact, be expedient to permit relatively large sting plunging amplitudes in favor of decreased static and dynamic sting interference. Such an approach stems from the fact that, unlike aerodynamic sting interference, sting plunging effects may readily be accounted for.

The sting plunging phenomenon has been treated analytically, alternatively as a 1-DOF linear,[2] 2-DOF linear,[1,4] or 1-DOF nonlinear problem.[1] Closed-form solutions to the equations for sting/model motion were derived which, in the linear cases, were of a fairly general nature. However, in spite of these efforts, it seems that 1) sting plunging corrections are as yet not universally applied, perhaps because they might have the appearance of being awkward to implement, and 2) the shift in the center of rotation of the model accompanying sting plunging oscillation, which has a potentially large effect on the values of the moment derivatives referred to the measurement center, appears to have been overlooked in most dynamic stability experiments to date.

In a recent paper, the present author[5] demonstrated that the equations due to Burt and Uselton[2], expressing the measured derivatives in terms of sting deflection constants, may be extended to a form equivalent to Ericsson's equations[1] for the effect of sting translational oscillation on the moment derivatives, and that both of these methods can be verified. An expression was then derived for the location of the effective axis of rotation for a model undergoing a pitch-plunge oscillation and subsequently used to simplify the equations for the correction of the moment derivatives.

In this paper, the available techniques are reviewed and the present analysis leading to expressions for the measured derivatives in terms of the location of the rotation center is summarized. The requirements for analysis of sting motion in high-α aircraft dynamic stability tests are examined and the application of the approach to pitch-oscillation tests of the Standard Dynamics Model (SDM)[6] is discussed.

Since, in general, the largest deflections would be likely to occur during oscillation in the pitch plane, the analysis is confined to the pitching derivatives. Nevertheless, the extension to the yaw derivatives should be feasible.

Evaluation of Existing Methods

Figure 1 illustrates the geometry of the model/sting system deflected under aerodynamic and inertial loads. The model angle of attack is

$$\alpha = \bar{\alpha} + \theta + \tan^{-1}[\dot{z}/(V_\infty \cos\bar{\alpha})] \tag{1}$$

Assuming linear aerodynamics and small oscillation amplitudes, the differential equations of motion for a model oscillating in the pitch plane, where $q = \dot{\theta}$, may be written as

$$I_y\ddot{\theta} + D\dot{\theta} + K\theta = M_{\ddot{\theta}}\ddot{\theta} + M_{\dot{\theta}}\dot{\theta} + M_\theta\theta + M_{\ddot{z}}\ddot{z} + M_{\dot{z}}\dot{z} + M_e(t)$$

$$m\ddot{z} + D_z\dot{z} + K_z z = Z_{\ddot{\theta}}\ddot{\theta} + Z_{\dot{\theta}}\dot{\theta} + Z_\theta\theta + Z_{\ddot{z}}\ddot{z} + Z_{\dot{z}}\dot{z} \tag{2}$$

Upon further simplification, a set of coupled 2-DOF equations having a solution of the following form may be obtained:

$$\theta = \Delta\theta_0 e^{i(\omega t - \Phi)}, \qquad z = \Delta z_0 e^{i\Phi_z}[\theta/\Delta\theta_0] \tag{3}$$

Review of Available Techniques

A literature survey revealed the existence of only three publications, namely, those due to Ericsson,[1] Burt and Uselton,[2] and Canu,[4] dealing explicitly with the analysis of sting motion in pitch damping tests.

2-DOF Linear Analysis—Canu[4]

Canu transformed the linearized 2-DOF equations, equivalent to Eq. (2), to a frame of reference for which the 2-DOF system reduces to a single-DOF system. In principle, the method appears to account correctly for the effects of sting plunging on the aerodynamic reactions and the oscillation axis. However, the viability of the method has not been demonstrated, at least as far as the paper reviewed is concerned. Moreover, although the method was developed for on-line data reduction, this advantage is negated by the requirement for testing at two different oscillation frequencies. More importantly, this involves the assumption that the pitch damping is independent of frequency, which cannot be justified in the general case.

2-DOF Linear Analysis—Ericsson[1]

In contrast, the merits of Ericsson's analysis were quite clear. Ericsson obtained a general solution of the form of Eq. (3), assuming that the sting deflection angle is small, i.e., $\theta = \theta_f$ (see Fig. 1). In order to obtain tractable closed-form equations, the analysis was then restricted to low-lift configurations, assuming that the aerodynamic and mechanical damping are negligible compared with the critical damping. In terms of wind-on and -off conditions, the derivatives are†

$$\bar{C}_{mq\dot{z}} = \bar{C}_{mq} - \frac{\Delta\zeta}{\Delta\theta}C_{m\alpha}(\bar{\omega}_{z0}^2 - \bar{\omega}^2)\left[1 + \bar{\omega}^2\left(\frac{\bar{C}_{Lq}}{C_{L\alpha}}\right)^2\right]^{-1/2}$$

$$\times\left[(\bar{\omega}_{z0}^2 - \bar{\omega}^2)^2 + \bar{\omega}^2\left(\delta_{z0} + \frac{C_{L\alpha}}{\mu}\right)^2\right]^{-1/2}$$

$$= \left(\frac{\Delta C_{m0}}{\Delta\theta\bar{\omega}}\sin\Phi\right)_{\text{off}}\Omega - \left(\frac{\Delta C_{m0}}{\Delta\theta\bar{\omega}}\sin\Phi\right)_{\text{on}}$$

$$C_{m\theta z} = C_{m\alpha}\left[1 - \left(\frac{\Delta\zeta}{\Delta\theta}\right)^2\bar{\omega}^2\frac{1 + (\mu\delta_{z0}/C_{L\alpha})}{1 + \bar{\omega}^2(\bar{C}_{Lq}/C_{L\alpha})^2}\right]$$

$$= \mu\kappa^2[(\bar{\omega}^2)_{\text{off}} - (\bar{\omega}^2)_{\text{on}}]$$

$$-\left(\frac{\Delta C_{m0}}{\Delta\theta}\cos\Phi\right)_{\text{on}} + \left(\frac{\Delta C_{m0}}{\Delta\theta}\cos\Phi\right)_{\text{off}} \tag{4}$$

An approximate form of Eqs. (4) is obtained for noncritical sting stiffness, when it can be assumed that $\delta_{z0} + C_{L\alpha}/\mu \ll 1$.

$$\bar{C}_{mq\dot{z}} = \bar{C}_{mq} - \frac{\Delta\zeta}{\Delta\theta}C_{m\alpha}\cos\Phi_z$$

$$C_{m\theta z} \approx C_{m\alpha}\left[1 - \bar{\omega}^2\left(\frac{\Delta\zeta}{\Delta\theta}\right)^2\right], \qquad \frac{\Delta\zeta}{\Delta\theta} = \frac{C_{L\alpha}}{\mu}\Big/|\bar{\omega}_{z0}^2 - \bar{\omega}^2|$$

$$\Phi_z \approx \cos^{-1}\left\{\frac{\bar{\omega}_{z0}^2 - \bar{\omega}^2}{|\bar{\omega}_{z0}^2 - \bar{\omega}^2|}\right\} \begin{matrix} = 0; & \bar{\omega}_{z0} > \bar{\omega} \\ = \pi; & \bar{\omega}_{z0} < \bar{\omega} \end{matrix} \tag{5}$$

†If the factor 2 is included in the nondimensional forms, e.g., $q\bar{c}/(2V_\infty)$, which is more common practice, corresponding factors of 2 will appear in the equations for C_{mq}.

Ericsson's entire analysis could be verified with the exception of one small discrepancy; the signs of Φ_z and the term $(\Delta\zeta/\Delta\theta)C_{m\alpha}\cos\Phi_z$ in Eqs. (5), as obtained from Eqs. (4), were reversed in Ericsson's paper but, of course, the final result was the same. This result is restricted to noncritical sting natural frequencies, and shows, for instance, that for subcritical sting stiffness, sting plunging will increase the measured damping and decrease the static stability. Implementation of Eqs. (5) could be based on the measurements of ω and ω_{z0}, or preferably, $\Delta\zeta$, $\Delta\theta$, and Φ_z, using appropriate sting instrumentation.

1-DOF Linear Analysis—Burt and Uselton[2]

Ignoring the lift forces at the outset, and including a mass unbalance term, Burt and Uselton obtained a linear pitching equation with a plunge equation of constraint. Then, again assuming the sting frequency is noncritical, the following derivatives were obtained:

$$M_q + M_{\dot\alpha} = \left[\left(\frac{M_\alpha}{V_\infty\cos\bar\alpha}\,\frac{\Delta z\cos\Phi_z}{\Delta\theta_f} + D - \frac{\Delta M_e\sin\Phi}{\omega\Delta\theta_f}\right) \div \left(1 + \frac{\Delta\theta_z}{\Delta\theta_f}\cos\Phi_z\right)\right]$$

$$-M_\alpha = K\left[\left(\frac{\omega}{\omega_{\theta 0}}\right)^2 - 1\right] + \left[\left(K\frac{\Delta\theta_z}{\Delta\theta_f}\cos\Phi_z - \delta K + \frac{\Delta M_e}{\Delta\theta_f}\cos\Phi + m x'_{cg}\omega^2\frac{\Delta z}{\Delta\theta_f}\cos\Phi_z\right)\Big/\left(1 + \frac{\Delta\theta_z}{\Delta\theta_f}\cos\Phi_z\right)\right] \qquad (6)$$

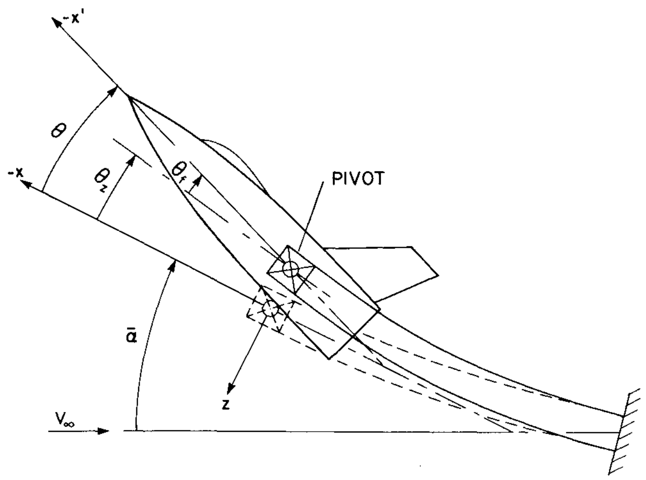

Fig. 1 Planar motion of sting-model system.

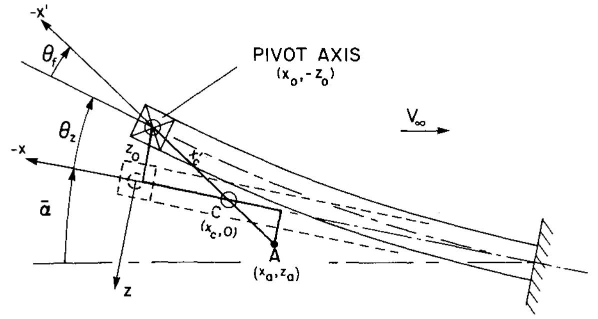

Fig. 2 Effective axis of oscillation.

A term in $M_{\dot\alpha}$ was neglected in the expression for M_α. Note that x'_{cg} is referred to the pivot axis.

Burt and Uselton expressed the sting oscillation parameters $\Delta\theta_z\cos\Phi_z/\Delta\theta_f$ and $\Delta z\cos\Phi_z/\Delta\theta_f$ in terms of the dynamic moments sensed by the sting and balance instrumentation and the sting deflection constants. For the case where the sting is not instrumented, expressions were derived for the sting oscillation parameters in terms of the dynamic force and moment at the pivot, assuming that static force data are available. Thus, in this case, the force equation of motion is introduced after the solution has been obtained.

1-DOF Nonlinear Analysis—Ericsson[1]

The linear results discussed previously could be quite inadequate in the presence of large oscillation amplitudes and significant nonlinearities. Ericsson addressed this problem for the case of a blunt cylinder-flare body, assuming that the lift is locally linear and the pitching moment nonlinear, but of a specific form. Except for the nonlinearity, the result obtained was subject to the same set of simplifying assumptions common to the linear methods and, therefore, may not have an advantage over the latter, other than in the special cases for which it can be formulated. It is not clear how the method can be generalized to aircraft configurations at high α, where the lift as well as the pitching-moment coefficients are likely to be highly nonlinear. These remarks notwithstanding, Ericsson's results provide a valuable demonstration that the effects of sting plunging can be larger in the nonlinear case.

Certain assumptions are common to all of the methods considered, including rigid mounting, structural damping small and independent of static loads and frequency, and low-lift and oscillation frequencies. A summary of restrictions is presented in Table 1. Only the 2-DOF analyses of Ericsson[1] and Canu[4] are applicable at near-critical sting frequencies and, with the exception of Ericsson's nonlinear analysis, the methods are based on linear aerodynamics and assumed small oscillation amplitudes. Canu's analysis determines the effective axis of oscillation, but this is not discussed in the other two papers.[1,2] The effective axis of rotation may be determined geometrically[7]; an appropriate method is introduced below.

Extension of the Analysis

Comparison of the 1- and 2-DOF Linear Methods

Assuming that $\Delta\theta_z \ll \Delta\theta_f$ (i.e., $\Delta\theta = \Delta\theta_f$), and that D is independent of static loads, the following relations may be derived from Eq. (6) for wind-on and -off conditions:

$$M_q + M_{\dot\alpha} - \frac{M_\alpha}{V_\infty\cos\bar\alpha}\,\frac{\Delta z\cos\Phi_z}{\Delta\theta} - D = -\left(\frac{\Delta M_e}{\omega\Delta\theta}\sin\Phi\right)_{\text{on}}$$

$$D = \left(\frac{\Delta M_e}{\omega\Delta\theta}\sin\Phi\right)_{\text{off}}\frac{\omega_{\text{off}}}{\omega_{\text{on}}} \qquad (7)$$

Then, writing $\bar M_q$ for $M_q + M_{\dot\alpha}$,

$$\bar M_{q\dot z} = \left(\frac{\Delta M_e}{\omega\Delta\theta}\sin\Phi\right)_{\text{off}}\frac{\omega_{\text{off}}}{\omega_{\text{on}}} - \left(\frac{\Delta M_e}{\omega\Delta\theta}\sin\Phi\right)_{\text{on}}$$

the following result is obtained in coefficient form:

$$\bar C_{mq\dot z} = \bar C_{mq} - \frac{\Delta\zeta}{\Delta\theta}C_{m\alpha}\cos\Phi_z \qquad (8)$$

which is in agreement with, although less complete than, Ericsson's approximate equation [Eqs. (5)]. Unlike its counterpart in Eqs. (5), $\Delta\zeta/\Delta\theta$ in Eq. (8) does not explicitly contain $C_{L\alpha}$.

Table 1 Summary of restrictions

Restrictions/ requirements	Canu[4]	Burt and Uselton,[2] Eqs. (6)	Ericsson[1] (linear), Eqs. (4)	Present method, Eqs. (14) and (15)	Ericsson[1] (nonlinear)
Eqs. of motion, DOFs	2	1	2	1 or 2[a]	1
Pitching moment	Linear	Linear	Linear	Linear	Nonlinear
Pitch damping	Independent of frequency	Negligible fraction of critical damping	Negligible fraction of critical damping	Negligible fraction of critical damping	Negligible fraction of critical damping
Dynamic lift	Small	Ignored	Small	Small[a]	Ignored
Model amplitude	Small, constant	Small, constant	Small, constant	Small, constant	Arbitrary, constant
Sting pitch amplitude	Negligible	Arbitrary, constant	Negligible	Negligible	Negligible
Sting frequency	Noncritical	Noncritical	Arbitrary	Arbitrary[a]	Noncritical
Mass offset	Negligible	Arbitrary	Negligible	Arbitrary	Negligible
Symmetry	Arbitrary shape	Arbitrary shape	Arbitrary shape	Arbitrary shape	Axisymmetrical
Axis transformation	Implicit	None	None	Implicit	None
Test/measurements	Two frequencies	Sting deflection parameters	Sting deflection parameters	Axis of rotation	Sting deflection parameters
Common restrictions	Low, linear lift; rigid support, structural damping small and independent of static loads; low reduced frequencies; constant sting plunge amplitude.				

[a]When applied to Eq. (4).

Similarly, if the variation of stiffness with applied force may be ignored, $\delta K=0$, and since $\omega_{\theta 0}^2=K/I_y$,

$$M_\alpha+\omega_{\text{on}}^2\left[I_y+mx'_{\text{cg}}\frac{\Delta z}{\Delta\theta}\cos\Phi_z\right]-K=-\left(\frac{\Delta M_e}{\Delta\theta}\cos\Phi\right)_{\text{on}}$$

$$\omega_{\text{off}}^2\left[I_y+mx'_{\text{cg}}\frac{\Delta z}{\Delta\theta}\cos\Phi_z\right]-K=-\left(\frac{\Delta M_e}{\Delta\theta}\cos\Phi\right)_{\text{off}} \tag{9}$$

Hence it may be shown that

$$C_{m\theta z}=C_{m\alpha}-2\mu\xi_{\text{cg}}(\bar\omega_{\text{off}}^2-\bar\omega_{\text{on}}^2)\frac{\Delta\zeta}{\Delta\theta}\cos\Phi_z$$

$$=2\mu\kappa^2(\bar\omega_{\text{off}}^2-\bar\omega_{\text{on}}^2)+\left(\frac{\Delta C_{me}}{\Delta\theta}\cos\Phi\right)_{\text{off}}-\left(\frac{\Delta C_{me}}{\Delta\theta}\cos\Phi\right)_{\text{on}} \tag{10}$$

Equation (10) reduces to the form of Eqs. (4) for the trivial case where $\xi_{\text{cg}}=0$ and $C_{Lq}=C_{L\alpha}=0$. Hence, it may be concluded that the methods due to Ericsson[1] and Burt and Uselton[2] will yield the same results at identical levels of approximation.

Note that the only contribution to $M_{\theta z}-M_\alpha$ is that due to unbalance. The fact that Eq. (10) is independent of aerodynamic forces is the consequence of neglecting such forces in the equation of motion. Moreover, the important conclusion drawn by Ericsson,[1] that sting plunging will, in general, lead to deficient static stability measurements, cannot be deduced from the analysis of Burt and Uselton.

Determination of the Effective Oscillation Axis

The axis of oscillation in a pitch-plunge motion relative to an inertial frame of reference may be derived from the sting-model geometry depicted in Fig. 2. Using the pivot axis as the origin, the location of the axis of rotation x'_c is obtained in terms of the angular deflection and the displacement z_a of a point x'_a on the model axis

$$\frac{z_0}{x'_c}=\frac{z_a}{x'_a-x'_c}=\sin(\theta_f+\theta_z) \tag{11}$$

whence

$$x'_c=x'_a-z_a/\sin(\theta_f+\theta_z) \tag{12}$$

The measurement of z_a could be obtained by means of an accelerometer mounted at point A. Assuming that $\theta_z\simeq 0$, which is perfectly reasonable when the sting is relatively rigid as in most dynamic stability testing, and since $\theta\simeq\theta_f$ is small, Eq. (11) yields

$$\frac{\zeta_0}{\theta}=\frac{\Delta\zeta_0}{\Delta\theta}=\xi_c \tag{13}$$

When the nondimensional location of the center of rotation given by Eq. (13) is introduced into Eqs. (5), the following simple form is obtained:

$$\bar C_{mq\dot z}=\bar C_{mq}\pm\xi_c C_{m\alpha};\quad \begin{matrix}\bar\omega_{z0}<\bar\omega\\ \bar\omega_{z0}>\bar\omega\end{matrix}$$

$$C_{m\theta z}=C_{m\alpha}[1-(\xi_c\bar\omega)^2] \tag{14}$$

Equations (14) do not involve any new assumptions since $\Delta\theta\simeq\Delta\theta_f$ is already implicit in Eqs. (5).

The application of Eqs. (14) requires a single measurement of ξ_c. As shown below, this parameter can be measured directly by optical means. Therefore, it is possible to apply approximate corrections for the effects of sting plunging, even when no sting instrumentation is available; however, the practicability of direct measurements of x'_c might be limited by experimental constraints. The restrictions associated with this approach are included in Table 1.

Normally, it should be possible to balance the model well enough that any residual mass offset might be ignored. When a large unbalance has to be contended with, the effects on $C_{m\alpha}$ might be estimated from the following expression derived from Eq. (10):

$$C_{m\theta z}=C_{m\alpha}\pm 2\mu\xi_{\text{cg}}\xi_c(\bar\omega_{\text{off}}^2-\bar\omega_{\text{on}}^2);\quad \begin{matrix}\bar\omega_{z0}<\bar\omega\\ \bar\omega_{z0}>\bar\omega\end{matrix} \tag{15}$$

Subsequently, the measured moment derivatives may be transformed to the reference center using standard relations for transformations between different axes.[8] However, this procedure is unsatisfactory in the case of the dynamic derivatives, introducing a requirement for additional tests at different centers of rotation.

Dynamic Testing of Aircraft Configurations

General Considerations

At low α, the assumptions of linear aerodynamics and low lift are justified when the methods discussed are applied to pitch-oscillation tests of aircraft configurations. However, these restrictions would tend to be violated at higher angles, particularly in the poststall domain, i.e., at angles beyond the maximum-lift angle of attack. Under these conditions translational acceleration and dynamic coupling forces and moments can become significant in addition to the basic nonlinearities of the lift and pitching moment.

In most cases, a 2-DOF solution valid for high-lift conditions could be adequate in the data reduction for direct derivatives; however, should nonlinear aerodynamics have to be incorporated, a numerical solution would be required. Inclusion of the two DOFs of the supporting sting in the data reduction equations suggests a Lagrangian analysis of a lumped parameter system as in the treatments described by Hanff and Orlik-Rückemann[9] and Haberman.[10] When the reactions in other DOFs are to be considered the complexity increases rapidly, resulting in a system of n simultaneous second-order differential equations[11] (where n is the number of DOFs). Thus, in any analysis of aerodynamic cross coupling, where a minimum of three degrees of freedom would be involved, or modeling significant nonlinearities, closed-form solutions and, therefore, on-line corrections for sting oscillation would be ruled out. The only recourse would then be to account for sting motion in the data reduction equations. However, the reliability of such approaches could yet be impaired by other effects that cannot be modeled easily. For instance, while in all of these methods it is assumed that $\bar\omega$, $\Delta\theta$, and $\Delta\zeta$ are constant for a particular test situation, this is quite far from reality in high-α aircraft tests, where the responses may be of a highly transient nature due to dynamic flow behavior. Such situations could, perhaps, be handled through real-time numerical integration to correct continuously the deflection vectors sensed by the balance.

The situation described is clearly complex, but, from a practical point of view, it is likely that the simplified equations described herein will still yield reasonable estimates of the sting plunging contributions to the measured derivatives. For supercritical sting frequencies the corrections are small (see below) and, therefore, it is expected that the corrections would be useful in most high-α pitch-oscillation tests of aircraft models. The limits of applicability should be determined experimentally. Where it is necessary to separate the q and $\dot\alpha$ effects, it might perhaps be feasible to obtain estimates of the effect of sting plunging on $C_{m\dot\alpha}$ using approximate equations such as those derived in Ref. 5.

Alleviation of the Sting Oscillation Problem

There is considerable scope for minimizing sting oscillations. First, consider the contribution of sting plunging to the damping moment. It follows from Eqs. (5) that

$$\frac{\bar C_{mq\dot z}-\bar C_{mq}}{\bar C_{mq}} = -\frac{C_{m\alpha}}{\bar C_{mq}}\,\frac{C_{L\alpha}}{\mu}\,\frac{1}{|\bar\omega_{z0}^2-\bar\omega^2|}\,\frac{\bar\omega_{z0}^2-\bar\omega^2}{|\bar\omega_{z0}^2-\bar\omega^2|} \qquad (16)$$

For a subcritical sting and stable model, the minimum damping contribution is achieved as $\bar\omega_{z0}\to 0$:

$$\frac{\bar C_{mq\dot z}-\bar C_{mq}}{\bar C_{mq}} = \frac{C_{m\alpha}}{\bar C_{mq}}\,\frac{C_{L\alpha}}{\mu\bar\omega^2} \qquad (17)$$

On the other hand, at an arbitrary supercritical sting frequency $\bar\omega_{z0}=c\bar\omega$, where $c>\sqrt{2}$; Eq. (16) yields

$$\frac{\bar C_{mq\dot z}-\bar C_{mq}}{\bar C_{mq}} = -\frac{C_{m\alpha}}{\bar C_{mq}}\,\frac{C_{L\alpha}}{\mu\bar\omega^2(c^2-1)} \qquad (18)$$

Therefore, the undamping contribution due to plunging of a supercritical sting is always smaller than the subcritical damping contribution, provided that $\bar\omega_{z0}>\sqrt{2}\bar\omega$. When $\bar\omega_{z0}>3\bar\omega$, which is fairly common in dynamic stability tests involving acceleration or cross-coupling derivative measurements, the ratio is smaller than 1/8. Thus, by designing for maximum sting stiffness compatible with permissible sting-model base-diameter ratios, it may be possible to obviate the need for sting plunging corrections, at least in the case of the direct derivatives.

Finally, it is noted here that the application of certain new concepts for dynamic stability testing[12] could, to all intents and purposes, eliminate sting oscillation in the pitch-yaw oscillation mode. The principle of orbital fixed-plane motion makes it possible, inter alia, to generate pure pitching and yawing motions. The static aerodynamic forces would then produce a fixed static deflection relative to body axes, since the angles of attack and sideslip are invariant and only the second-order dynamic forces and moments would tend to contribute to sting oscillation. The implementation of this concept could, therefore, be advantageous in the determination of cross-coupling derivatives due to pitching and yawing.

Oscillatory Testing About the Effective Axis

The notion of measuring aircraft stability parameters about the effective axis of rotation has a broader significance than in the above context. In aircraft poststall maneuvers involving simultaneous, large-amplitude pitching and plunging/heaving, the effective center of rotation may be very different from the center of mass (cg) as illustrated schematically in Fig. 3.[13] The dynamic parameters should then preferably be determined about the effective center of rotation rather than about the cg as the resulting flow about the model would then be more representative of the actual aircraft motion in flight. In this way, an experimental 2-DOF pitch-heave motion can be reduced to a simple fixed-axis rotation. The aerodynamic parameters are subsequently transformed to the reference center.

Practical Application

SDM Pitch-Oscillation Tests

In dynamic stability testing of the Standard Dynamics Model (SDM), the approach has been to maximize the sup-

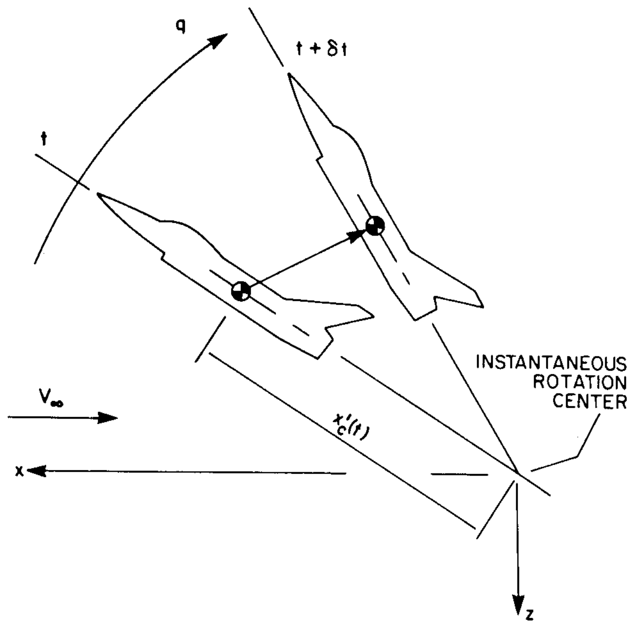

Fig. 3 Effective center of rotation in flight.

port rigidity consistent with the comments in the previous section, yet maintaining sting-model geometries acceptable from a point of view of aerodynamic sting interference. By virtue of high sting stiffnesses and supercritical sting frequencies, $\bar{\omega}_{z0}/\bar{\omega} \simeq 3$, the sting-oscillation amplitudes were small and concomitant effects on the measured derivatives could be expected to be small.

In the test series under consideration, the SDM was oscillated in three different modes: about the body pitch and yaw axes and the aerodynamic pitch axis (i.e., in the plane of the total angle of attack). The complete set of dynamic and static moment derivatives, including the direct, cross, and cross-coupling derivatives, were determined in each mode, at Mach 0.6 and α up to 45 deg in some cases, and for angles of sideslip $\beta = 0$ and 5 deg.[6]

Measurements of the Axis of Rotation

The method used for locating the effective axis of rotation was one of visually locating a nontranslating point on the model with the help of a stroboscopic light source directed through the test-section window. The reading was taken from one of two scales enscribed on the fuselage just below and above the wing root. In the procedure used, the strobe frequency was adjusted to be slightly off the wind-on oscillation frequency so that a point on the model could be followed with the naked eye.

For an "optimum" setting of the strobe frequency, the accuracy of the measurements under wind-off conditions was found to be ± 0.5 mm, but for low-α wind-on conditions the accuracy was closer to ± 1 mm. At high α, the precision deteriorated somewhat but the main source of uncertainty was apparently the variation of x_c with time as a result of flow unsteadiness. In this context it should be noted that, while the pitch-oscillation waveform $\alpha(t)$ was quite steady and, in fact, almost sinusoidal even at very high α, the $z(t)$ variation was more erratic; short intervals of periodic motion alternating with periods in which the sting motion appeared to be dominated by random excitation. Under such conditions (i.e., at $\alpha \geq 30$ deg) observation of the nontranslating point became difficult, and the location of the oscillation axis had to be obtained as the average of several readings (as many as five). Since the intervals of periodic motion observed correspond to $\Delta\zeta$, $\Delta\theta$ constant, the corrections may still be reliable under such unsteady conditions. This may not be true of measurements obtained from inertial instrumentation.

Results

Pitch-Oscillation Tests

The measured values of ξ_c are presented in Fig. 4 as a function of α; the dashed lines represent the band of experimental uncertainty for $\beta = 0$. The direction of the shift was aft ($\xi_c > 0$). The magnitude of the ξ_c readings was confirmed in a bench test where the alternating aerodynamic loads were simulated by means of an unbalance mass attached to the model nose. The size of the lumped mass was derived from the variation in normal force over the angular excursion, viz, ± 1.0 deg. The simulated axis shift, $x_c' = 4$ mm, was of the correct order of magnitude.

The measured direct derivatives appear in Fig. 5. Applying Eqs. (14) to these data it is found that the corrections for translational effects are very small, with $|\bar{C}_{m q \dot{z}} - \bar{C}_{mq}| \leq 0.04$ and $C_{m\theta z} \simeq C_{m\alpha}$. On the other hand, the transformation of the derivatives to the reference center located ξ_c from the axis of rotation results in non-negligible corrections. The exact values of the corrections are dependent on derivatives such as $C_{L\alpha}$ and $C_{Lq} + C_{L\dot{\alpha}}$, which were not measured here, but the corrections to $C_{m\alpha}$ and $C_{mq} + C_{m\dot{\alpha}}$ could be estimated to be of the order of -0.2 and 1, respectively.

Other Planes of Measurements

In the case of pitching about the aerodynamic pitch axis, the shift in the oscillation axis was found to be roughly equivalent, or slightly smaller than, its counterpart in oscillation about the body pitch axis contained in Fig. 4. On the other hand, in the yaw oscillation mode, sting oscillations could be detected in the plane of oscillation for the $\beta = 5$ deg case, but the amplitude and resultant axis shift were too small to be measurable. Secondary effects, i.e., sting lateral oscillations in the plane perpendicular to the plane of the primary, forced motion were negligible or not detectable even at $\beta = 5$ deg in all three experimental modes.

These observations lend credence to the assumption that, when a supercritical sting is used, corrections for sting plunging need to be applied only in the reduction of the pitch-oscillation derivatives. The major effect to be accounted for is the shift in the center of oscillation.

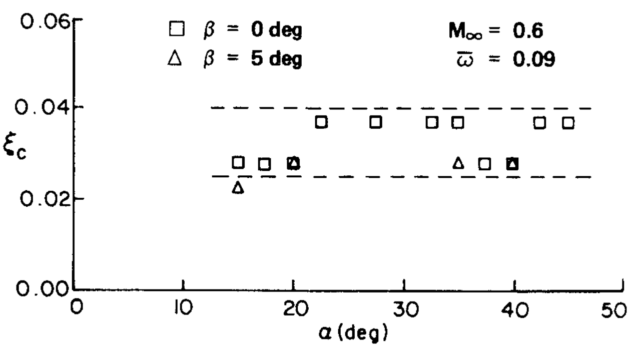

Fig. 4 Effective rotation center in SDM pitch-oscillation tests.

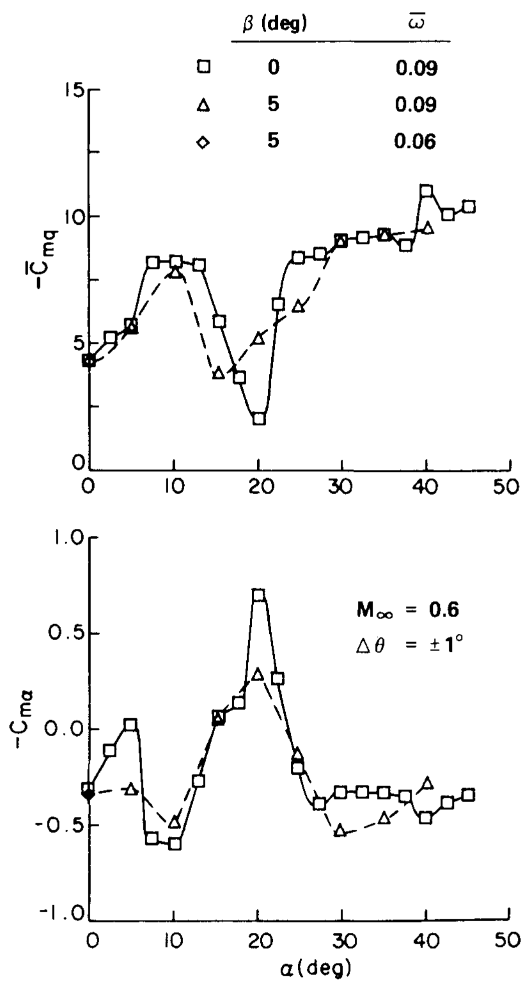

Fig. 5 SDM direct pitch stability derivatives.[6]

Conclusions

The contributions of sting plunging to the moment derivatives measured in small-amplitude pitch/yaw oscillation tests are, in general, reliably accounted for in Ericsson's linear analysis for low-lift configurations.[1] The approximate form of these equations is particularly useful in on-line data reduction. The more simplified analysis due to Burt and Uselton[2] could also yield acceptable results for low-lift configurations and noncritical sting frequencies ($\bar{\omega}_{z0} \geq 3\bar{\omega}$). These two methods reduce to the same result at identical levels of approximation.

Expressions were derived for the location of the effective axis of rotation of a model executing pitch-plunge oscillations, and a simple optical technique was introduced for determining the location of this axis. The readings can be taken to an accuracy of within ± 1 mm, provided that the axis does not fall beyond the extremities of the model.

It was shown that the introduction of the effective axis of oscillation into Ericsson's approximate equations yields a simple form that may be implemented without the use of sting instrumentation. The correction for translational effects on the measured derivatives can then be accomplished in addition to their transformation to the reference center by means of this procedure.

It was demonstrated that, under conditions encountered in general, aircraft pitch-oscillation testing, the shift in the rotation center can produce the dominant effect resulting from sting plunging oscillation. When the technique was applied to oscillatory tests of the Standard Dynamics Model (SDM), it was found that

1) The axis shift yielded significant effects on the derivatives, which had to be accounted for.

2) Corrections for translational effects on the derivatives were relatively small, and were required only in the case of the pitch-oscillation derivatives.

3) The use of a supercritical sting has the distinct advantage of minimizing sting plunging effects.

4) The results obtained by means of this simple technique would otherwise require inertial instrumentation in more than one plane of the balance.

No theory is available for the estimation of sting plunging effects on cross-coupling derivative measurements. However, as pointed out previously, the out-of-plane effects are small or negligible, at least under conditions corresponding to the SDM experiment.

Nevertheless, it should be noted that in high-α testing, most of the assumptions implicit in these techniques tend to be violated and, therefore, it is recommended that further research be undertaken to obtain a more general description of the phenomenon, even if its ultimate form is not conducive to straightforward corrections. The altogether greater complexity of data reduction procedures based on multi-DOF models of the sting-balance-model system, including nonlinear aerodynamics, possibly as an extension of Ericsson's nonlinear technique, must be considered necessary in future experiments designed to obtain accurate measurements of stability derivatives at high α.

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